

Time-Space Hybrid Dynamic Integration Algorithm for Inelastic Earthquake Time-History Analysis of Building Structures

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Abstract

To solve the convergence problem of the building structures under the strong nonlinear condition, this paper develops a series of hybrid algorithms for strong nonlinear analysis of the building structures, which combine the advantages of explicit and implicit algorithms. In the time-domain hybrid algorithm, the switch between the implicit algorithm and the explicit is decided by the occurrence of iteration non-convergence when the implicit algorithm is used for the analysis of the whole building structure. And in the space hybrid algorithm, the whole structure is divided into several floor areas and interface areas to decouple the independent dynamic balance equation of each floor area by predicting the dynamic responses of interface, so that the proper dynamic integral algorithm of each area could be confirmed through its real state. The time-space hybrid algorithm, using the occurrence of the implicit algorithm iteration non-convergence as the switch criterion for the dynamic algorithm in the time domain and referring to the displacement difference vector of the iteration failure moment, can determine the area in the state of strong nonlinear and automatically partition the building structure according to the freedom number, then switch to the space hybrid algorithm to calculate the current time step. The hybrid dynamic algorithm realized in the self-development finite element analysis platform presents the superiority in solving the strong nonlinear problems over the Newmark algorithm when it is applied to the nonlinear time-history analysis of strong earthquake for building structures.

Keywords: Hybrid dynamic algorithm, Earthquake, Inelastic analysis, Building structures.

1. INTRODUCTION

The whole dynamic process simulation can effectively reveal the catastrophic behavior of the building structure under the collapse level earthquake, but the computational efficiency problem of the explicit algorithm and the convergence problem of the implicit algorithm constrain the simulation. Therefore, the study of hybrid dynamic integration algorithm is meaningful as suggested by Cai et al. (2012). The earliest implementation of the hybrid algorithm is usually used in the stamping springback simulation which achieves the switching of different algorithms in the time domain. Jung and Yang (1998) use the implicit algorithm in the initial calculation and the algorithm is switched to the explicit algorithm when the number of iterations of the implicit algorithm exceeds the limit after the impact contact. Narasimhan and Lovell (1999) employ the explicit algorithm in the stamping process and the implicit algorithm in the springback process and other processes. The research about the algorithm automatic switching derives from Ludovic et al. (2002), who takes the CPU time-consuming of algorithm as the selection basis of the implicit and explicit algorithm. The space hybrid algorithm is mainly used for the analysis of fluid-solid coupling problem began with Belytschko and Mullen (1978). And different integration schemes including asynchronous algorithm of explicit-explicit partition and asynchronous algorithm of implicit-explicit partition as suggested by Liu and Belytschko (1982) are adopted in

different partitions. Smolinski (1996) proposes an explicit sub-cycle algorithm that the time step ratio can be a non-integer for structural dynamic calculations. Then Daniel (1997) proposes an implicit sub-cycle algorithm based on Newmark's discrete format, whose numerical stability is superior to the explicit sub-cycle algorithm. Based on the explicit and implicit Newmark algorithm, Zhang and Jin (2014) propose an arbitrary explicit and implicit hybrid asynchronous algorithm. It could be concluded that there is certain research basis in the time-domain and space hybrid algorithm, and the time domain hybrid algorithm has been applied into some commercial FE software. The space hybrid algorithm, at the same time, can realize the multi-partition synchronous analysis and show good stability. However, these studies don't focus on the problem of the strong nonlinear analysis of the building structure. This paper focuses on the hybrid dynamic algorithm applied to the nonlinear analysis of building structures, and the time hybrid dynamic algorithm, space hybrid dynamic algorithm and time-space hybrid dynamic algorithm (hereinafter referred to as THDA, SHDA and TSHDA) are used to carry out the seismic inelastic time-history analysis respectively. Considering the consistency of dynamic analysis parameters, the explicit Newmark algorithm and implicit Newmark algorithm are selected to implement the hybrid dynamic algorithm. Based on the finite element analysis platform DUT2014 as suggested by Fu et al. (2015), three kinds of hybrid dynamic algorithms are achieved.

2. THE TIME-DOMAIN HYBRID DYNAMIC ALGORITHM

THDA is a mixture of implicit and explicit algorithms in the time domain, which employs the implicit algorithm to calculate with large time step when the implicit algorithm can converge, and uses the explicit algorithm to analyze with subdivided time step when the implicit algorithm can't converge. The schematic diagram of THDA is shown in Figure 1.

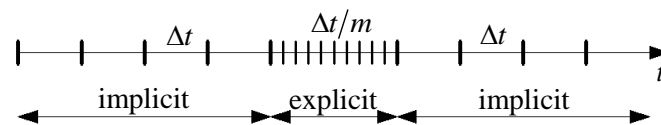


Figure 1. Schematic diagram of the time-domain hybrid algorithm

The following is the detailed description of the THDA calculation process. (1) The implicit Newmark algorithm is used to start the time-history analysis. (2) In the process of analysis, whether the implicit algorithm converges is set as switching criteria. (3) The response vectors of the structure are reset to the vectors at the beginning of the time step. (4) The time step of the explicit Newmark algorithm should be subdivided. (5) The load calculation of the explicit Newmark algorithm: in the cycle of subdivided m steps (the loop variable is i), the calculation method of the full load at i step is $P_0 + (i/m) \times (\Delta p + F_p)$, where P_0 is the full seismic load at $i = 0$, Δp is the incremental seismic load at current time step, and F_p is the imbalance force generated in the previous step. (6) After the time step analyzed by explicit Newmark algorithm, the analysis algorithm is switched back to the implicit Newmark algorithm to continue calculation.

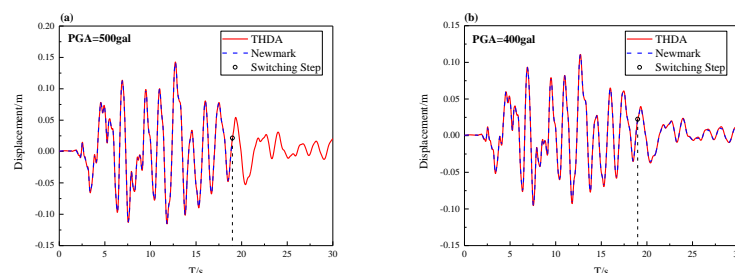


Figure 2. Comparison diagram of time-history displacement of vertex

The Newmark algorithm and THDA are used to carry out the earthquake time-history analysis of a 6-

story frame structure. The structure has a span of 5.1m in both X and Y direction, the story height is 3m, the column size is 0.6m×0.6m, the beam size is 0.5m×0.3m, and the floor thickness is 0.12m. The implicit Newmark algorithm is used to do the analysis, where $\gamma=1/2$, $\beta=1/4$, the analysis time step is taken as 0.01s, the convergence tolerance is 1.0E-4, and the maximum number of iteration is 20. When the PGA level reaches 500 gal, the implicit Newmark algorithm does not converge at 1899 step. Then the structure is re-analyzed by THDA, where the explicit time step subdivision m is 10000. It can be seen from Figure 2(a) that the displacement calculation results are not deviated at the switching step and the results are not divergent at the follow-up time steps. In order to verify the accuracy of THDA, the implicit Newmark algorithm and THDA are used to do the time-history analysis of this structure where PGA=400gal, and the implicit Newmark algorithm is forced to switch to the explicit Newmark algorithm at step 1899. The comparison diagram of the complete displacement time-history curve of Newmark and THDA in Figure 2(b) shows that the calculation results of THDA are credible.

3. THE SPACE HYBRID DYNAMIC ALGORITHM

Under the earthquake load excitation, most areas in the building structure are in the state of weak nonlinearity, and only some local areas enter the state of strong nonlinearity which may lead to the convergence problem. This paper has developed SHDA shown in Figure 3, which divides the entire building structure into multiple sections according to the state of nonlinearity. It takes the explicit algorithm with small time step to analyze strong nonlinear regions, and the implicit algorithm with large time step to analyze weak nonlinear regions, and couples the interaction between partitions through the interface.

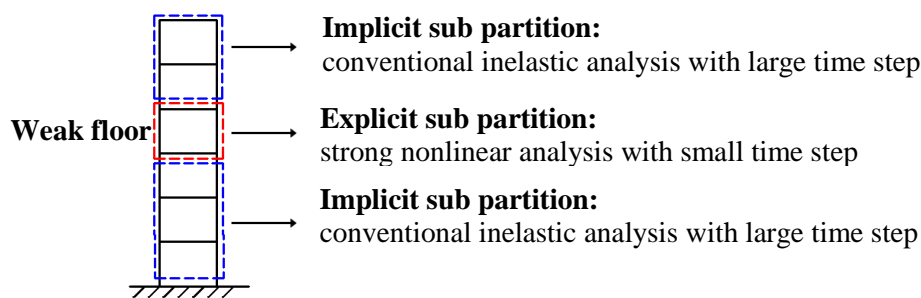


Figure 3. Schematic diagram of the space hybrid algorithm

The system is divided into a number of substructures, where **I** and **E** represent the degree of freedom of the partition and the interface, respectively. Then the system dynamic equilibrium Equation 1 where can be expressed as Equation 2 which can deduce partition dynamic balance Equation 3 and interface dynamic balance Equation 4 through the equation rearranging.

$$\mathbf{M}\mathbf{x}'' + \mathbf{C}\mathbf{x}' + \mathbf{K}\mathbf{x} = \mathbf{P} \quad (1)$$

$$\begin{bmatrix} \mathbf{M}_{II} & \mathbf{M}_{IE} \\ \mathbf{M}_{EI} & \mathbf{M}_{EE} \end{bmatrix} \begin{bmatrix} \mathbf{x}_I'' \\ \mathbf{x}_E'' \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{II} & \mathbf{C}_{IE} \\ \mathbf{C}_{EI} & \mathbf{C}_{EE} \end{bmatrix} \begin{bmatrix} \mathbf{x}_I' \\ \mathbf{x}_E' \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{II} & \mathbf{K}_{IE} \\ \mathbf{K}_{EI} & \mathbf{K}_{EE} \end{bmatrix} \begin{bmatrix} \mathbf{x}_I \\ \mathbf{x}_E \end{bmatrix} = \begin{bmatrix} \mathbf{P}_I \\ \mathbf{P}_E \end{bmatrix} \quad (2)$$

$$\mathbf{M}_{II}\mathbf{x}_I'' + \mathbf{C}_{II}\mathbf{x}_I' + \mathbf{K}_{II}\mathbf{x}_I = \mathbf{P}_I - \mathbf{M}_{IE}\mathbf{x}_E'' - \mathbf{C}_{IE}\mathbf{x}_E' - \mathbf{K}_{IE}\mathbf{x}_E \quad (3)$$

$$\mathbf{M}_{EE}\mathbf{x}_E'' + \mathbf{C}_{EE}\mathbf{x}_E' + \mathbf{K}_{EE}\mathbf{x}_E = \mathbf{P}_E - \mathbf{M}_{EI}\mathbf{x}_I'' - \mathbf{C}_{EI}\mathbf{x}_I' - \mathbf{K}_{EI}\mathbf{x}_I \quad (4)$$

In the above equations, \mathbf{x} , \mathbf{x}' and \mathbf{x}'' are the displacement, velocity and acceleration vector of the system respectively, \mathbf{K} , \mathbf{M} and \mathbf{C} are the stiffness, mass and damping matrix of the system respectively, \mathbf{P} is the external load vector of the system, the subscript II and I represent the matrix and vector of the implicit sub partition, the subscript EE and E represent the matrix and vector of the explicit sub partition, and the IE and EI represent the matrix Coupled with the implicit and explicit sub partition.

If the responses of system at i step are known, the responses at $i+1$ step can be solved as the following calculation steps: (1) The responses of interface at $i+1$ step are predicted through assuming that the acceleration of the interface at this time step is constant. (2) The right-hand term for each partition dynamic balance Equation 3 can be obtained. (3) Each partition is calculated independently using the appropriate algorithm and the analysis step, and then the responses of each partition at $i+1$ step can be obtained. (4) The responses of interface at $i+1$ step can be got through taking the responses of each partition into Equation 4. (5) The predicted responses of interface should be corrected. (6) After the responses of system at $i+1$ time step are obtained, the responses at i time step, the system matrix \mathbf{K} , \mathbf{C} , and the restoring force vector \mathbf{f}_s should be updated to carry out the calculation at next time step.

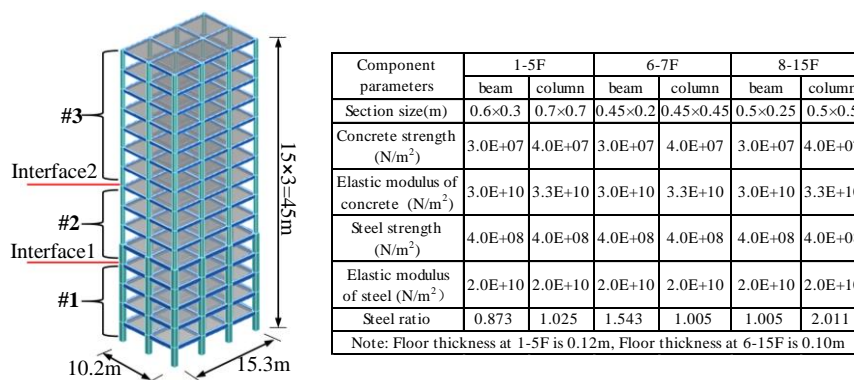


Figure 4. Schematic diagram of a 15-story reinforced concrete frame structure

In the inelastic time-history analysis of the structure shown in Figure 4, beam-column members are modelled with fiber-beam-column element, floors are modelled by the layered shell element, failure and removal of the beam-column element are considered and the norm of the displacement increment vector should be less than the convergence tolerance. The input is a seismic wave which lasts for 30s. The implicit Newmark algorithm is used to analyze with analysis parameters $\gamma=1/2$, $\beta=1/4$, step size of $1/100s$, convergence tolerance of 0.001, maximum iterations of 20. The PGA level of seismic wave will gradually increase from 100gal to 700gal, when PGA is greater than 400gal, Newmark algorithm will fail to obtain complete time history analysis results owing to failure of iteration. In order to solve the convergence problem, SHDA is used to do the analysis. As shown in above, the #2 partition adopts the explicit algorithm with time step of $1/50000s$; the #1 and #3 partitions adopt the implicit algorithm with time step of $1/100s$. Figure 5 shows that the results of the two algorithms are in good agreement in the time domain that the implicit Newmark algorithm can converge and SHDA can carry on the stronger nonlinear analysis compared with the Newmark algorithm.

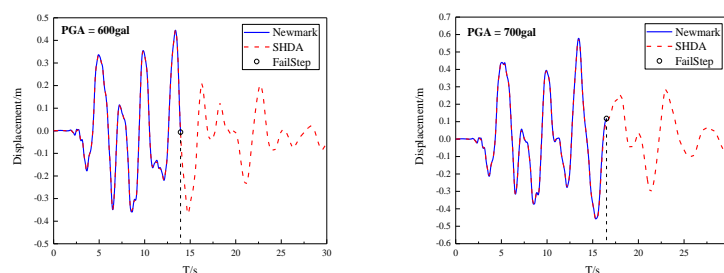


Figure 5. Comparison diagram of inelastic analysis results of the Newmark and SHDA

4. THE TIME-SPACE HYBRID DYNAMIC ALGORITHM

THDA is limited by the stability condition, and SHDA needs to determine the location of the weak position of the structure in advance. Therefore, combined with THDA and SHDA, the time-space hybrid dynamic algorithm (TSHDA) is proposed. As shown in Figure 6, the implicit algorithm with large time step Δt is used to calculate the whole structure at first, when the implicit algorithm doesn't converge, it is possible to determine the strong non-linear part to partition the structure into different parts. Then the current time step is analysed by SHDA that the strong non-linear part is analysed by the explicit algorithm with time step of $\Delta t/m$, and the time step of other sub-partitions is still Δt . After the current time step is calculated, the algorithm is switched back to the implicit algorithm to calculate the whole structure until the end of the seismic wave.

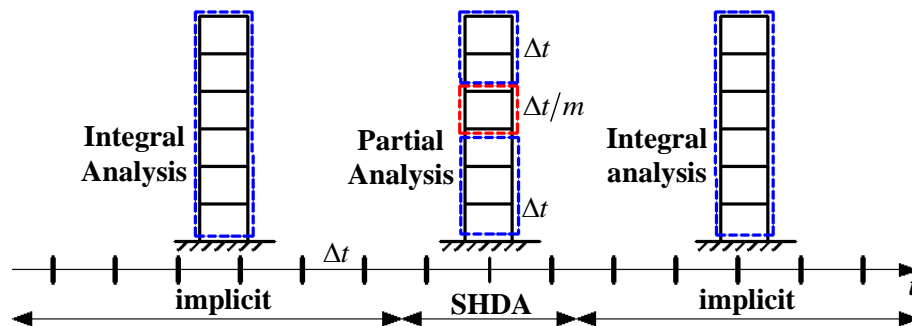


Figure 6. Schematic diagram of the time-space hybrid dynamic algorithm

The detailed calculation flow of the space-time hybrid dynamic algorithm is shown in Figure 7. On the one hand, TSHDA improves the computational efficiency of SHDA and on the other hand enhances the versatility of the hybrid algorithm. For the general nonlinear analysis, TSHDA uses the implicit Newmark algorithm to complete the time-history analysis in the whole time domain. As for the strong nonlinear analysis, the Newmark algorithm tends to fail in some time steps, and SHDA can be used to solve it in those time steps.

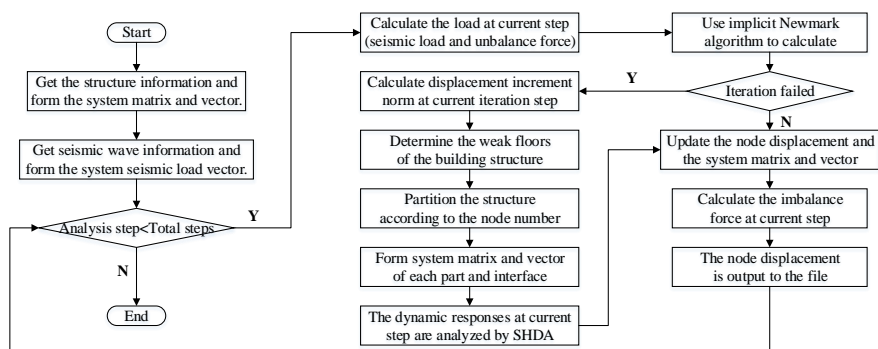


Figure 7. The calculation flow diagram of the time-space hybrid dynamic algorithm

As for the structure shown in Figure 4 with the same PGA level and dynamic parameters in SHDA analysis, the vertex time history displacement and algorithm switching time step obtained by TSHDA are shown in Figure 8. It can be seen that the calculation results of SHDA and TSHDA are basically identical, and the error is within 0.03m. When PGA=600gal, the calculation time of SHDA is 12346s compared with 817s of TSHDA; When PGA=700gal, the SHDA is 12886s compared with 1449s of TSHDA. It can be seen that TSHDA is more efficient.

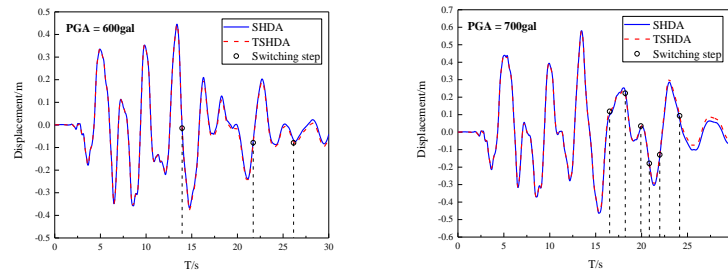


Figure 8. Comparison chart of inelastic analysis results of the SHDA and TSHDA

5. CONCLUSION

It could be obtained from the reinforced concrete frame structures' inelastic time-history analysis that the hybrid algorithm proposed in this paper overcomes the convergence problem of implicit algorithm and the computational efficiency problem of explicit algorithm, which could be used to do the strong nonlinear analysis when the structure suffers collapse scale earthquake. THDA is more suitable for the small structures subject to its stable condition. The problem of pre-specified partition of SHDA makes it suitable for the structure with definite stiffness distribution characteristics. TSHDA achieves mixing of algorithms in different time stages and regions, and has larger time step and higher efficiency, so it is fit for the analysis of the large scale structure.

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